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Title: **Addressing Stiff Physics Problems Using a Novel Implicit Solver that has a Computational Complexity Equivalent to Explicit Methods**

Abstract:

In 1859 we experienced our first recorded massive solar flare to strike the earth, known as the Carington event. The way we identified this event was the sudden combustion of telegraph machines. The flare dumped a large amount of charged particles into the Earth's magnetosphere, where energetic particles were trapped due to the Lorenz force. As the particles bounced back and fourth between the polls, the current of the charged particles inductively coupled with the telegraph wires, driving sufficient current into the telegraph machines to cause combustion.

Today if such an event happened, it would cause massive damage to our power grid across large sections of the country all at once. Conservative estimates indicate that it would take us 10 years to recover from such an event, not to mention the high risk associated with nuclear power plants. An obvious solution is a decentralized power grid which would localize damage, but the next best solution is to be able to predict such an event and shut off the power grid on a temporary basis. However, in the case of a solar flare, we have a tight time constraint, the first burst of x-rays reach the earth in 9 minutes, followed by the first bunch of relativistic electrons at the three hour mark, and finally around three days the bulk of the flare carrying the heavy particles reaches the earth. This means there is at most a 72 hour window to make an accurate prediction followed by decisions on an action. This is the field of space weather modeling.

Ideally one would use multi-fluid models coupled with Maxwell's equations. However, if done with an explicit method, this system would need 10^{10} time steps and is intractable in the 72 hour window. Alternatively, Magnetohydrodynamics, which is a single fluid model of plasmas, captures the essential physics and can make the 72 hour window. To model these type of systems, it is important to include the non-ideal terms. These terms include a resistive term, Hall term, electron inertial term and the electron pressure term. The addition of this physics makes these equations far stiffer than the ideal MHD equations, introducing a range of much faster time scales that are not in the ideal MHD equations. The goal of this work is to create a method that can address stiff time scales in such a way that the method can take a CFL time step of ideal MHD with the computational cost of an explicit method. This is done by making use of the Method of Lines Transpose. The eventual goal is to speed up space weather modeling frameworks by a factor of a 1000, making it possible to run 500 to 1000 scenarios in the time we currently run one prediction of solar flare. This opens the door for a range of improvements including uncertainty quantification.

MOLT starts by employing your favorite explicit time discretization. Next the spatial derivatives are approximated using an expansion of convolution integrals with a particular kernel. The expiation is a convergent approximation where each additional term increases the order of accuracy in time. The particular expansion has the effect of making linear PDE's provably unconditionally stable with explicit time stepping, and behaves unconditionally stable for non-linear PDE's. The integrals are computed in $O(N)$ using a fast convolution method with that comparable with most explicit finite difference methods. We have applied this method a range of PDE's, including degenerate advection diffusion and the Hamilton Jacobi equation, but hear we use the method to solve the auxiliary equation in constrained transport. As you will see, this equation contains the stiff terms and by addressing it with MOLT, the non-ideal MHD solver will be able to capture the stiff terms while letting us take a CFL of one for the ideal MHD equations. These our the first steps in achieving out goals of decreasing the computation time while increasing the usefulness of space wether modeling.

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